The Inversion Invariance of the Lorentz Transformation

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It is shown that the fundamental quantity v of the inverse transformation does not turn out to be negative.

1. The Inversion Invariance

Usually, frames of observation that are not mutually symmetric, such as two equidirectional reference systems S and S', show the inertial process of uniform motion towards each other asymmetrically distorted to an observer. This then yields a Galilean transformation of the form

$$x' = -v\,t + x, \qquad (1)$$

that is not invariant against an interchange of the two systems ("dash-change"), since in the inverse transformation

$$x = v\,t + x', \qquad (2)$$

the fundamental quantity v appears with the opposite sign.

Because of the total spatio-temporal symmetry of the inertial process, however, the claim for formal equality of transformation and inverse transformation, as a primary instrument of control, is indispensably and absolutely necessary.

In order to ensure the inversion invariance of the Galilean transformation, in view of the total symmetry of the inertial process, also the frames of observation have to be symmetrized. For surveying the two spaces moving against each other one has to use, e.g., a right-handed system S and a left-handed system S'. Then the x-axes of the two systems point symmetrically towards each other, and the Galilean transformation takes, because of

$$x + x' = v\,t. \qquad (3)$$

the inversion invariant form

$$x' = v\,t - x \quad \text{as well as} \quad x = v\,t - x'. \qquad (4)$$

In (4) the dashes can be moved to and fro without the fundamental quantity changing its sign. With the realization of inversion invariance we have gained a new operational method that allows us to demonstrate on the basis of the symmetry of the process that a transformation originating from a symmetric process is without contradiction in itself.

2. The Inversion-invariant Lorentz Transformation

In the Lorentz transformation of Lorentz, Poincaré and Einstein,

$$x' = \gamma(-v\,t + x) \quad \text{and} \quad t' = \gamma\left(t - \frac{v}{c^2}x\right), \quad (5)$$

that belongs to a similar the same inertial process as the Galilean transformation, we face the same asymmetric transformation structure. Again two equidirectional systems S and S' are employed. The asymmetric distortion caused by that arrangement excludes inversion invariance and again makes the fundamental quantity appear with the opposite sign in the inverse transformation

$$x = \gamma(v\,t' + x') \quad \text{and} \quad t = \gamma\left(t' + \frac{v}{c^2}x'\right). \quad (6)$$

In a substantial part of the extensive protest literature to the theory of special relativity the sign asymmetry of the orthodox Lorentz transformation is one reason for attack. While in the case of the Galilean transformation the transition from the form (1)–(2) to the inversion invariant form (4) can be easily seen, establishing the inversion invariance of the Lorentz transformation is not so transparent, as non-intuitiveness from Lorentz contraction and time dilatation plays a role there. If, however, we use a right-handed and a left-handed system for the usual spherical derivation we readily arrive at the inversion invariant Lorentz transformation

$$x' = \gamma(v\,t - x) \quad \text{and} \quad t' = \gamma\left(t - \frac{v}{c^2}x\right), \quad (7)$$

as well as at the formally identical inverse transformation

$$x = \gamma(v\,t' - x') \quad \text{and} \quad t = \gamma\left(t' - \frac{v}{c^2}x'\right). \quad (8)$$

And with that it is effectively ensured that the Lorentz transformation is without contradiction in this respect. Furthermore, the inversion invariance exhibits an immaculate beauty of the Lorentz transformation that one could expect in virtue of the perfect symmetry of the inertial process.